COMP3151/9154

Foundations of Concurrency

Shared Variable Proof Methods, Hardware-Assisted Critical Sections

Johannes Åman Pohjola
CSE, UNSW
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In this lecture, we will introduce a formal proof method for verifying safety properties, and apply it to a new kind of critical section solution that relies on hardware support.

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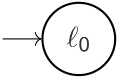

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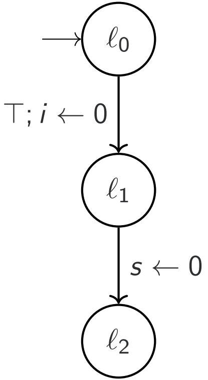

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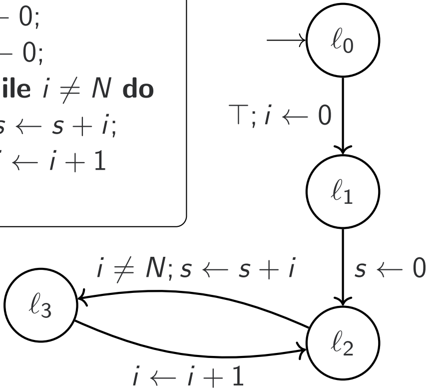

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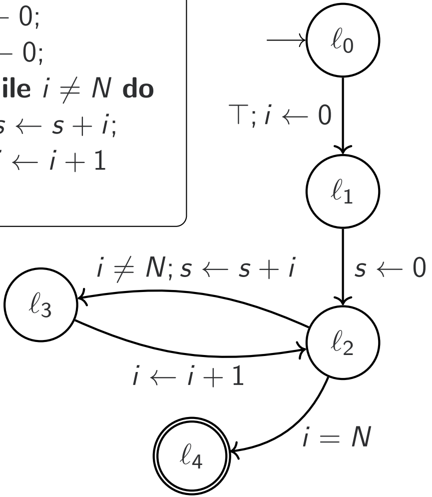

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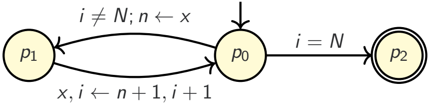
Johannes will now demonstrate on the previous example

Adding Concurrency



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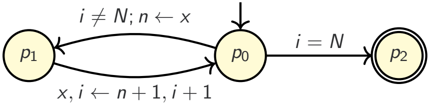

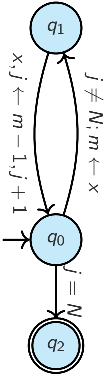

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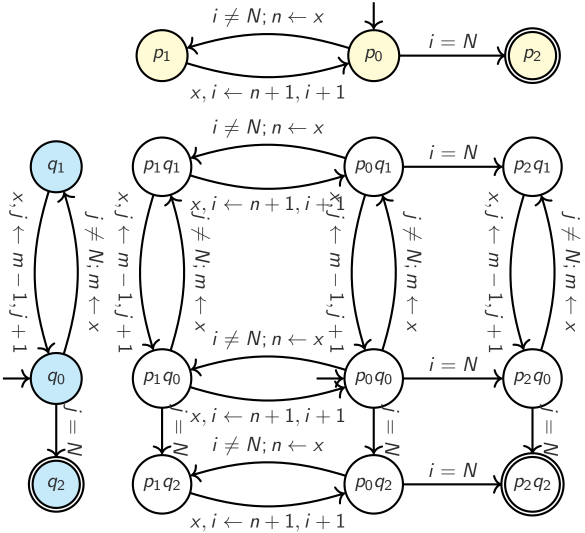

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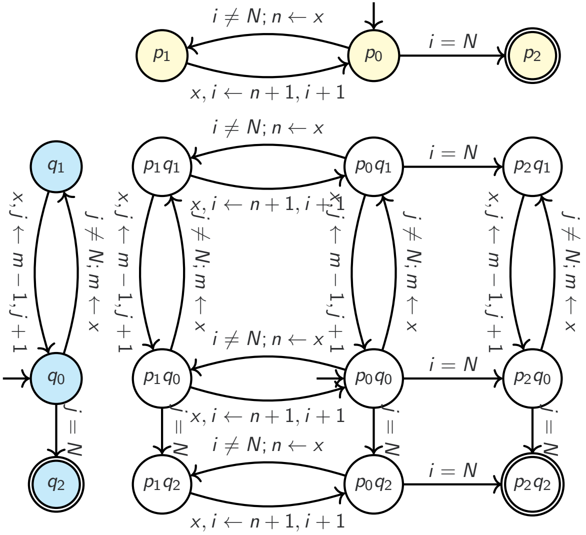

Parallel Composition

Given two processes P and Q with transition diagrams (L_P, T_P, s_P, t_P) and (L_Q, T_Q, s_Q, t_Q), the *parallel composition* of P and Q, written $P \parallel Q$ is defined as (L, T, s, t) where:

- $L = L_P \times L_Q$
- $s = s_P s_Q$
- $t = t_P t_Q$
- $p_i q_i \xrightarrow{g;f} p_j q_i \in T$ if $p_i \xrightarrow{g;f} p_j \in T_P$
- $p_i q_i \xrightarrow{g;f} p_i q_j \in T$ if $q_i \xrightarrow{g;f} q_j \in T_Q$

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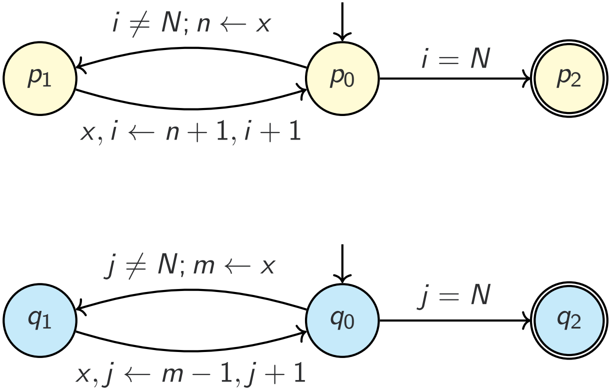
Susan Owicki's solution

Define inductive assertion networks for P and Q separately. By proving some non-interference properties derive an inductive network for $P \parallel Q$ automatically. This means we won't have to draw that large product diagram!

Owicki-Gries Method



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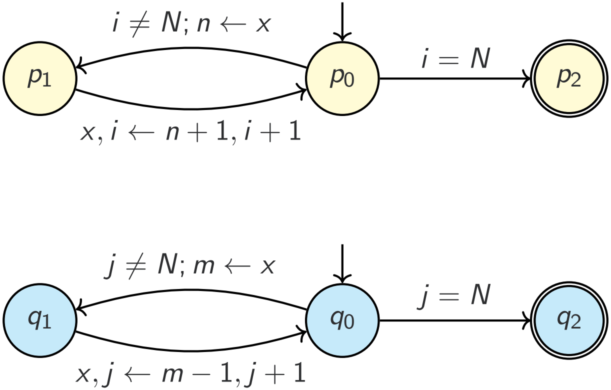

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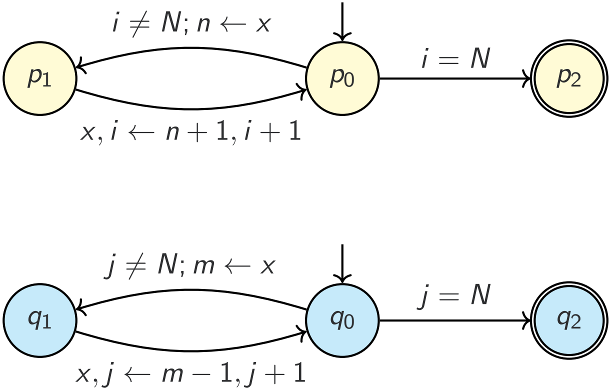

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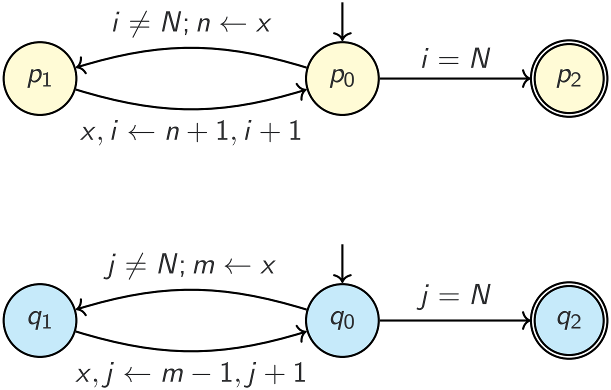

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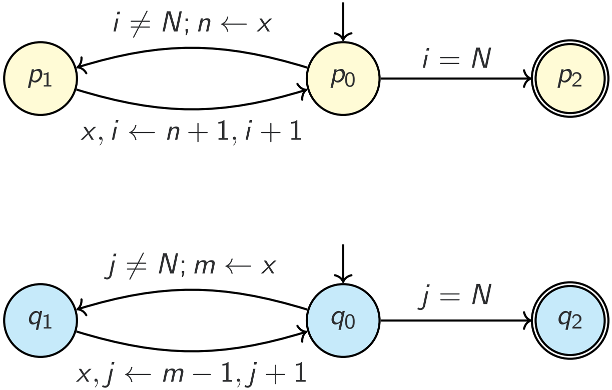

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Owicki-Gries reduces that to $2 + n \cdot k \cdot (1 + (n-1) \cdot m)$ — merely quadratic in n.

Proving Mutual Exclusion

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Manna-Pnueli Algorithm			
integer wantp, wantq $\leftarrow 0, 0$			
	forever do		**forever do**
p_1	*non-critical section*	q_1	*non-critical section*
p_2	**if** wantq $= -1$ **then** wantp $\leftarrow -1$ **else** wantp $\leftarrow 1$	q_2	**if** wantp $= -1$ **then** wantq $\leftarrow 1$ **else** wantq $\leftarrow -1$
p_3	**await** wantq $\neq$ wantp	q_3	**await** wantq $\neq -$wantp
p_4	*critical section*	q_4	*critical section*
p_5	wantp $\leftarrow 0$	q_5	wantq $\leftarrow 0$

Note: The p_2 and q_2 steps are one atomic step!

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bit common ← 1	
bit tp ← 0	bit tq ← 0
forever do	**forever do**
p_1 *non-critical section*	q_1 *non-critical section*
repeat	**repeat**
p_2 `XC`(tp, common)	q_2 `XC`(tq, common);
p_3 **until** tp = 1	q_3 **until** tq = 1
p_4 **critical section**	q_4 **critical section**
p_5 `XC`(tp, common)	q_5 `XC`(tq, common)

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$$\begin{array}{rcl}\mathcal{I} & \equiv & (\mathsf{common} \oplus \mathsf{tp} \oplus \mathsf{tq}) = 1 \land (P@p_4 \Rightarrow \mathsf{tp} = 1) \land \\ & & (Q@q_4 \Rightarrow \mathsf{tq} = 1) \land \neg(\mathsf{common} = \mathsf{tp} \land \mathsf{common} = \mathsf{tq})\end{array}$$

Where $\oplus$ is `xor`. Note: $\mathcal{I}$ is false at $p_4 q_4$. So if this invariant is preserved we have mutex.

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Lets prove mutual exclusion for `XC`!

What now?

- You now have all you need to complete Assignment 0 (warm-up), due Monday the 20th.
- Next week: We will examine some more sophisticated critical section solutions for n processes.
- We may also learn about *semaphores*, time permitting!.